CBCS SCHEME

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21MATME41

Fourth Semester B.E. Degree Examination, June/July 2023
Complex Analysis, Probability and Linear Programming

Time: 3 hrs. Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Use of Statistical Tables is permitted.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notation, derive the Cauchy's Riemann equations in the polar form. **(06 Marks)**

b. If f(z) is regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$. **(07 Marks)**

c. Determine the analytic function whose real part is $u = e^x(x\cos y - y\sin y)$. **(07 Marks)**

OR

2 a. With usual notation, derive the Cauchy's Rieman in the Cartesian form. **(07 Marks)**

b. Show that $f(z) = \left(r+\frac{k^2}{r}\right)\cos\theta + i\left(r-\frac{k^2}{r}\right)\sin\theta$, $r \neq 0$ is regular function $z = re^{i\theta}$, find $f'(z)$. **(06 Marks)**

c. Find the analytic function whose real part is $u = \log\sqrt{x^2+y^2}$. **(07 Marks)**

Module-2

3 a. Discuss the transformation $w = z^2$. **(06 Marks)**

b. State and prove the Cauchy's integral formula. **(07 Marks)**

c. Find the bilinear transformation which maps the point z = 0, 1, ∞ into the points w = −5, −1, 3 respectively. **(07 Marks)**

OR

4 a. Find the bilinear transformation which maps the points z = 1, i, −1 to w = i, 0, −1. **(06 Marks)**

b. Verify Cauchy's theorem for the integral of z^2 over the boundary:
(i) Along the st-line z = 0 to z = 3 + i
(ii) Along the curve made up to two line segments, one from z = 0 to z = 3 and another from z = 3 to z = 3 + i. **(07 Marks)**

c. Evaluate $\oint_c \frac{e^2 z}{(z-1)^2(z-2)}dz$ where $c : |z| = 3$ **(07 Marks)**

Module-3

5 a. A random variable x has the following probability following various values of x:

x	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

i) Find K (ii) Evaluate $P(x < 6)$ (iii) $P(3 < x \le 6)$ **(06 Marks)**

b. Find the mean and standard deviation of the Poisson distribution. **(07 Marks)**

c. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probability of
(i) No error during a micro second (ii) One error (iii) Atleast one error
(iv) Two error (v) Almost two error **(07 Marks)**

OR

6 a. Find the constant K such that $f(x) = \begin{cases} Kx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$ is a p.d.f. Also compute:
(i) $P(1 < x < 2)$ (ii) $P(x \le 1)$ (iii) $P(x > 1)$ (iv) mean and variance **(06 Marks)**

b. The marks of 1000 students in an examination follows a normal distribution with mean 70 and S.D. is 5. Find the number of students whose marks will be:
(i) Less than 65 (ii) More than 75 (iii) Between 65 and 75 **(07 Marks)**

c. The length of telephone conversation in booth has been an exponential distribution and found an average, to be 5 minutes. Find the probability that a random call made from this booth (i) ends less than 5 min (ii) In between 5 and 10 min. **(07 Marks)**

Module-4

7 a. Using Simplex method
Maximize $Z = 5x_1 + 3x_2$
Subject to the constraints $x_1 + x_2 \le 2$
$5x_1 + 2x_2 \le 10$
$3x_1 + 8x_2 \le 12$
$x_1, x_2 \ge 0$ **(10 Marks)**

b. Solve the following L.P.P. by the Simplex method
Minimize $Z = x_1 - 3x_2 + 3x_3$
Subject to the constraints $3x_1 - x_2 + 2x_3 \le 7$
$-4x_1 + 3x_2 + 8x_3 \le 10$
$2x_1 + 4x_2 \ge -12$
$x_1, x_2, x_3 \ge 0$ **(10 Marks)**

OR

8 a. Define the following terms a linear programming problem, basic solution, basic feasible solution, optimal solution, artificial variable of an L.P.P. **(10 Marks)**

b. Use the two phase method to
Minimize $Z = 7.5x_1 - 3x_2$
Subject to the constraints $3x_1 - x_2 - x_3 \ge 3$
$x_1 - x_2 + x_3 \ge 2$
$x_1, x_2, x_3 \ge 0$ **(10 Marks)**

Module-5

9 a. Find the initial basic feasible solution by Vogel's method to the following transportation problem. **(10 Marks)**

		A	B	C	D	Availability
Source	I	21	16	25	13	11
	II	17	18	14	23	13
	III	32	27	18	41	19
	Requirement	6	10	12	15	43

b. Four jobs are to be done on four machines. The cost (in rupees) of producing i^{th} job on the j^{th} machine is given below:

Machine

Jobs		M_1	M_2	M_3	M_4
	J_1	15	11	13	15
	J_2	17	12	12	13
	J_3	14	15	10	14
	J_4	16	13	11	17

Assign the jobs to the different machines so as to minimize the total cost. **(10 Marks)**

OR

10 a. A company has three cement factorize located in cities 1, 2, 3 which supply cement to four projects located in town 1, 2, 3, 4. Each plant can supply 6, 1, 10 truck loads of cement daily respectively and the daily cement requirements of the project and respectively 7, 5, 3, 2, truck loads. The transport cost per truck load of cement (in hundred of rupees) from each plant to each project for the following:

Project sites

Factories		1	2	3	4
	1	2	3	11	7
	2	1	0	6	1
	3	5	8	15	9

Determine the optimal distribution for the company so as to minimize the total transportation cost. **(10 Marks)**

b. Solve the following transportation problem:

To

From							
	9	12	9	6	9	10	5
	7	3	7	7	5	5	6
	6	5	9	11	3	11	2
	6	8	11	2	2	10	9
	4	4	6	2	4	2	22

(10 Marks)

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